NNLO Jet Cross Sections by Subtraction

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Motivation

Why jets at NNLO?

Hadronic jets occur frequently in final states of high energy particle collisions.

Because of large production cross sections, jet observables can be measured with high statistical accuracy; can be ideal for precision studies.

Examples include measurements of:

- α_s **from jet rates and event shapes in** $e^+e^- \to$ **jets;**
- **gluon PDFs and** α_s **from** $2+1$ **jet production in DIS;**
- **PDFs in single jet inclusive,** $V+$ **jet in** pp **(or** $p\bar{p}$**) collisions.**

Often, relevant observables measured with accuracy of a few % or better.

Theoretical predictions with same level of accuracy necessary. This usually requires NNLO corrections.

What is a subtraction scheme?

We know that IR singularities cancel according to the KLN theorem between real and virtual quantum corrections at the same order in perturbation theory, for sufficiently inclusive (IR safe) observables.

Example (simple residuum subtraction)

$$\boxed{\sigma = \int_0^1 \mathrm{d}\sigma^{\mathrm{R}}(x) + \sigma^{\mathrm{V}}\,,} \qquad \textbf{where} \qquad \begin{aligned} &\mathrm{d}\sigma^{\mathrm{R}}(x) = x^{-1-\epsilon} S(x)\,, \\ &S(0) = S_0 < \infty\,, \\ &\sigma^{\mathrm{V}} = S_0/\epsilon + F\,. \end{aligned}$$

Define the counterterm $\mathrm{d}\sigma^{\mathrm{R,A}}(x) = x^{-1-\epsilon} S_0$. **Then**

$$\begin{aligned} \sigma &= \int_0^1 \left[\mathrm{d}\sigma^{\mathrm{R}}(x) - \mathrm{d}\sigma^{\mathrm{R,A}}(x)\right]_{\epsilon=0} + \left[\sigma^{\mathrm{V}} + \int_0^1 \mathrm{d}\sigma^{\mathrm{R,A}}(x)\right]_{\epsilon=0} \\ &= \int_0^1 \left[\frac{S(x) - S_0}{x^{1+\epsilon}}\right]_{\epsilon=0} + \left[\frac{S_0}{\epsilon} + F - \frac{S_0}{\epsilon}\right]_{\epsilon=0} \\ &= \int_0^1 \frac{S(x) - S_0}{x} + F \end{aligned}$$

The last integral is finite, computable with standard numerical methods.

In a rigorous mathematical sense, the cancellation of both kinematical singularities and ϵ-poles must be local. I.e. the counterterm must have the following general properties

- **must match the singularity structure of the real emission cross section pointwise, in d dimensions**
- **its integrated form must be combined with the virtual cross section explicitly, before phase space integration; ϵ-poles must cancel point by point**

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The construction should be universal (i.e. process and observable independent)

- to avoid tedious adaptation to every specific problem
- the integration of counterterms can be performed once and for all
- the IR limits of QCD (squared) matrix elements are universal, so a general construction should be possible

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Different specific choices of the counterterm correspond to different IR subtraction schemes (CS dipole, FKS, antenna,. . .).

Why a new subtraction scheme at NNLO?

Dipole subtraction (Catani, Seymour)

- ✔ the counterterms are completely local
- ✔ the construction is fully explicit for a general process
- ✘ faces fundamental difficulties when going to NNLO

Antenna subtraction (Gehrmann-De Ridder, Gehrmann, Glover; Weinzierl)

$q_\perp$ subtraction (Catani, Grazzini; Cieri, Ferrera, de Florian)

Sector decomposition (Binoth, Heinrich; Anastasiou, Melnikov, Petriello)

This scheme (Del Duca, GS, Trócsányi)

Why a new subtraction scheme at NNLO?

Dipole subtraction (Catani, Seymour)

Antenna subtraction (Gehrmann-De Ridder, Gehrmann, Glover; Weinzierl)

- ✔ successfully applied to $e^+e^- \to 3$ jets
- ✔ complete analytical integration of antennae is tractable
- ✘ the counterterms are not fully local
- ✘ cannot cut factorized phase space

$q_\perp$ subtraction (Catani, Grazzini; Cieri, Ferrera, de Florian)

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Antenna subtraction (Gehrmann-De Ridder, Gehrmann, Glover; Weinzierl)

$q_\perp$ subtraction (Catani, Grazzini; Cieri, Ferrera, de Florian)

- ✔ exploits universal behaviour of $q_\perp$ distribution at small $q_\perp$
- ✔ numerically efficient implementation possible
- ✘ applicable only to the production of colourless final states in hadron collisions

Sector decomposition (Binoth, Heinrich; Anastasiou, Melnikov, Petriello)

This scheme (Del Duca, GS, Trócsányi)

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$q_\perp$ subtraction (Catani, Grazzini; Cieri, Ferrera, de Florian)

Sector decomposition (Binoth, Heinrich; Anastasiou, Melnikov, Petriello)

- ✔ dispenses with the subtraction method, but conceptually very simple
- ✔ first method to yield physical cross sections

- ✘ cancellation of ϵ-poles numerical
- ✘ can it handle complicated final states?

This scheme (Del Duca, GS, Trócsányi)

Why a new subtraction scheme at NNLO?

Dipole subtraction (Catani, Seymour)

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This scheme (Del Duca, GS, Trócsányi)

- ✔ very algorithmic construction
- ✔ has the advantages of dipole scheme: counterterms are explicitly general and fully local
- ✔ can cut factorised phase space: important for efficiency

- ✘ analytical integration of the counterterms requires computing many new integrals (of high dimension) but can be done once and for all

Subtraction at NNLO

What is needed to define a subtraction scheme?

To define a subtraction scheme, three problems must be addressed

1. **Matching of limits: the known IR factorization formulae must be written in such a way, that the overlapping soft/collinear singularities can be disentangled in order to avoid multiple subtraction.**

$$\mathbf{A}_1 |\mathcal{M}_{m+1}^{(0)}|^2 = \sum_i \left[\sum_{i \neq r} \frac{1}{2} \mathbf{C}_{ir} + \mathbf{S}_r - \sum_{i \neq r} \mathbf{C}_{ir} \mathbf{S}_r \right] |\mathcal{M}_{m+1}^{(0)}|^2$$

2. **Extension over PS: the IR factorization formulae valid in the strict soft/collinear limits have to be defined over the full PS. This requires the introduction of appropriate mappings of momenta that respect factorization and the (delicate) cancellation of IR singularities**

$$\{p\}_{m+1} \xrightarrow{r} \{\tilde{p}\}_m : \quad \mathrm{d}\phi_{m+1}(\{p\}_{m+1}; Q) = \mathrm{d}\phi_m(\{\tilde{p}\}_m; Q)[\mathrm{d}p_{1,m}]$$
$$\{p\}_{m+2} \xrightarrow{r,s} \{\tilde{p}\}_m : \quad \mathrm{d}\phi_{m+2}(\{p\}_{m+2}; Q) = \mathrm{d}\phi_m(\{\tilde{p}\}_m; Q)[\mathrm{d}p_{2,m}]$$

3. **Integration: the counterterms have to be integrated over the phase space of the unresolved parton(s).**

Specific issues at NNLO

- Matching is cumbersome if done in a brute force way. However, an efficient solution that works at any order in PT is known.
- Extension is very delicate. Among other constraints, the counterterms for singly-unresolved real emission must have universal IR limits, which is not guaranteed by QCD factorization.
- Choosing the counterterms such that integration is (relatively) easy generally conflicts with the delicate cancellations in the various limits.

The NNLO cross section

Consider the NNLO correction to a generic *m*-jet observable

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{VV}}_{m} J_{m}\,.$$

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Doubly-real

- $\mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2}$
- **Tree MEs with $m+2$-parton kinematics**
- **kin. singularities as one or two partons unresolved: up to $\mathrm{O}(\epsilon^{-4})$ poles from PS integration**
- **no explicit ϵ poles**

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Real-virtual

- $\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1}$
- **One-loop MEs with $m+1$-parton kinematics**
- **kin. singularities as one parton unresolved: up to $\mathrm{O}(\epsilon^{-2})$ poles from PS integration**
- **explicit ϵ poles up to $\mathrm{O}(\epsilon^{-2})$**

The NNLO cross section

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- **Tree MEs with $m+2$-parton kinematics**
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- $\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1}$
- **One-loop MEs with $m+1$-parton kinematics**
- **kin. singularities as one parton unresolved: up to $\mathrm{O}(\epsilon^{-2})$ poles from PS integration**
- **explicit ϵ poles up to $\mathrm{O}(\epsilon^{-2})$**

Doubly-virtual

- $\mathrm{d}\sigma^{\mathrm{VV}}_{m} J_{m}$
- **One- and two-loop MEs with *m*-parton kinematics**
- **kin. singularities screened by jet function: PS integration finite**
- **explicit ϵ poles up to $\mathrm{O}(\epsilon^{-4})$**

The NNLO counterterms

Rewrite the NNLO correction as

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m}$$

$$= \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_2}_{m+2} J_m - \left[\mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_{12}}_{m+2} J_m \right] \right\}$$

$$+ \int_{m+1} \left\{ \left[\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} \right] J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV},\mathrm{A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] J_m \right\}$$

$$+ \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left[\mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_2}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_{12}}_{m+2} \right] + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV},\mathrm{A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] \right\} J_m$$

The NNLO counterterms

Rewrite the NNLO correction as

$$
\begin{aligned}
\sigma^{\mathrm{NNLO}} &= \int_{m+2} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m} \\
&= \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \left[\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \right] \right\} \\
&+ \int_{m+1} \left\{ \left[\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right] J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] J_m \right\} \\
&+ \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left[\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} \right] + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] \right\} J_m
\end{aligned}
$$

1. $\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2}$ **regularizes the doubly-unresolved limits of** $\mathrm{d}\sigma^{\mathrm{RR}}_{m+2}$

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Rewrite the NNLO correction as

$$\sigma^{\rm NNLO} = \int_{m+2} \mathrm{d}\sigma^{\rm NNLO}_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\rm NNLO}_{m+1} + \int_{m} \mathrm{d}\sigma^{\rm NNLO}_{m}$$

$$= \int_{m+2} \left\{ \mathrm{d}\sigma^{\rm RR}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left[\mathrm{d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right] \right\}$$

$$+ \int_{m+1} \left\{ \left[\mathrm{d}\sigma^{\rm RV}_{m+1} + \int_1 \mathrm{d}\sigma^{\rm RR,A_1}_{m+2} \right] J_{m+1} - \left[\mathrm{d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$+ \int_{m} \left\{ \mathrm{d}\sigma^{\rm VV}_{m} + \int_2 \left[\mathrm{d}\sigma^{\rm RR,A_2}_{m+2} - \mathrm{d}\sigma^{\rm RR,A_{12}}_{m+2} \right] + \int_1 \left[\mathrm{d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

1. $\mathrm{d}\sigma^{\rm RR,A_2}_{m+2}$ **regularizes the doubly-unresolved limits of** $\mathrm{d}\sigma^{\rm RR}_{m+2}$
2. $\mathrm{d}\sigma^{\rm RR,A_1}_{m+2}$ **regularizes the singly-unresolved limits of** $\mathrm{d}\sigma^{\rm RR}_{m+2}$

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Rewrite the NNLO correction as

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m}$$

$$= \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_2}_{m+2} J_m - \left[\mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_{12}}_{m+2} J_m \right] \right\}$$

$$+ \int_{m+1} \left\{ \left[\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} \right] J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV},\mathrm{A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] J_m \right\}$$

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3. $\mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_{12}}_{m+2}$ **accounts for the overlap of** $\mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2}$ **and** $\mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_2}_{m+2}$

The NNLO counterterms

Rewrite the NNLO correction as

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m}$$

$$= \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \left[\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \right] \right\}$$

$$+ \int_{m+1} \left\{ \left[\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right] J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] J_m \right\}$$

$$+ \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left[\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} \right] + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

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4. $\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1}$ **regularizes the singly-unresolved limits of** $\mathrm{d}\sigma^{\mathrm{RV}}_{m+1}$

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Rewrite the NNLO correction as

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} \mathrm{d}\sigma_{m+2}^{\mathrm{NNLO}} + \int_{m+1} \mathrm{d}\sigma_{m+1}^{\mathrm{NNLO}} + \int_{m} \mathrm{d}\sigma_{m}^{\mathrm{NNLO}}$$

$$= \int_{m+2} \left\{ \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2} J_m - \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} J_{m+1} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}} J_m \right] \right\}$$

$$+ \int_{m+1} \left\{ \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right] J_{m+1} - \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1} + \boxed{\left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right)^{\mathrm{A}_1}} \right] J_m \right\}$$

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1. $\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2}$ **regularizes the doubly-unresolved limits of** $\mathrm{d}\sigma_{m+2}^{\mathrm{RR}}$
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4. $\mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1}$ **regularizes the singly-unresolved limits of** $\mathrm{d}\sigma_{m+1}^{\mathrm{RV}}$
5. $\boxed{(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1})^{\mathrm{A}_1}}$ **regularizes the singly-unresolved limit of** $\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1}$

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$$= \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_2}_{m+2} J_m - \left[\mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_{12}}_{m+2} J_m \right] \right\}$$

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4. $\mathrm{d}\sigma^{\mathrm{RV},\mathrm{A}_1}_{m+1}$ **regularizes the singly-unresolved limits of** $\mathrm{d}\sigma^{\mathrm{RV}}_{m+1}$
5. $(\int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2})^{\mathrm{A}_1}$ **regularizes the singly-unresolved limit of** $\int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2}$

General features

- The counterterms are based on IR limit formulae.
- The counterterms are given completely explicitly for any process without coloured particles in the initial state. (The extension to hadronic processes is known explicitly to NLO.)
- The counterterms are fully local in colour $\otimes$ spin space: no need to consider the colour decomposition of real emission matrix elements; azimuthal correlations correctly taken into account in gluon splitting; can check explicitly that the ratio of the sum of counterterms to the real emission cross section tends to unity in any IR limit.
- It is straightforward to implement a cut on the factorized phase spaces. Thus, during PS integration, in any given PS point only a (small) subset of all subtraction terms needs to be explicitly evaluated. This is a large gain in efficiency.

Integrating the counterterms

Integrated counterterms

Counterterm	Types of integrals	Done
$\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1}$	**tree level singly-unresolved**	✔
$\int_1 \mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1}$	**one-loop singly-unresolved**	✔
$\int_1 (\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1})^{\mathrm{A}_1}$	**tree level iterated singly-unresolved (1)**	✔
$\int_2 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}}$	**tree level iterated singly-unresolved (2)**	✔
$\int_2 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2}$	**tree level iterated doubly-unresolved**	✘

Integrated counterterms

Counterterm	Types of integrals	Done
$\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1}$	tree level singly-unresolved	✔
$\int_1 \mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1}$	one-loop singly-unresolved	✔
$\int_1 (\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1})^{\mathrm{A}_1}$	tree level iterated singly-unresolved (1)	✔
$\int_2 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}}$	tree level iterated singly-unresolved (2)	✔
$\int_2 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2}$	tree level iterated doubly-unresolved	✘

Phase space integrals - an example

Example (collinear-double collinear counterterm)

Among many others, in $\mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2}$ we find the following collinear-double collinear counterterm

$$\mathcal{C}_{kt}\mathcal{C}^{(0)}_{ir;kt} = (8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon})^2 \frac{1}{s_{kt}} \frac{1}{\widehat{s}_{ir}} \langle \mathcal{M}^{(0)}_m(\{\tilde{p}\})|P^{(0)}_{f_k f_t}(z_{t,k};\epsilon)P^{(0)}_{f_i f_r}(\widehat{z}_{r,i};\epsilon)|\mathcal{M}^{(0)}_m(\{\tilde{p}\})\rangle$$
$$\times (1-\alpha_{kt})^{2d_0-2m(1-\epsilon)}(1-\widehat{\alpha}_{kt})^{2d_0-2m(1-\epsilon)}\Theta(\alpha_0-\alpha_{kt})\Theta(\alpha_0-\widehat{\alpha}_{ir})$$

The set of m momenta, $\{\tilde{p}\}$, is obtained by an iterated mapping, and leads to an exact factorization of phase space

$$\{p\}_{m+2} \xrightarrow{\mathsf{C}_{kt}} \{\hat{p}\}_{m+1} \xrightarrow{\mathsf{C}_{\hat{i}\hat{r}}} \{\tilde{p}\} : \quad \mathrm{d}\phi_{m+2}(\{p\};Q) = \mathrm{d}\phi_m(\{\tilde{p}\};Q)[\mathrm{d}\widehat{p}_{1,m}][\mathrm{d}p_{1,m+1}]$$

We must then compute

$$\int [\mathrm{d}\widehat{p}_{1,m}][\mathrm{d}p_{1,m+1}]\mathcal{C}_{kt}\mathcal{C}^{(0)}_{ir;kt} \equiv \left[\frac{\alpha_{\mathrm{s}}}{2\pi}S_\epsilon\left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 \mathrm{C}_{kt}\mathrm{C}^{(0)}_{ir;kt}\mathbf{T}^2_{kt}\mathbf{T}^2_{ir}|\mathcal{M}^{(0)}_m(\{\tilde{p}\})|^2$$

where $\mathrm{C}_{kt}\mathrm{C}^{(0)}_{ir;kt} \equiv \mathrm{C}_{kt}\mathrm{C}^{(0)}_{ir;kt}(\widetilde{x}_{kt}, \widetilde{x}_{ir}, \epsilon, \alpha_0, d_0)$ is a kinematics dependent function.

Example (collinear-double collinear "master integrals")

Using some explicit parametrization of the factorized phase space measures, $[\mathrm{d}\widehat{p}_{1,m}]$ and $[\mathrm{d}p_{1,m+1}]$, we find we can express $\mathrm{C}_{kt}\mathrm{C}^{(0)}_{ir;kt}$ as a linear combination of the following "master integrals"

$$\mathcal{I}^{(4)}_{\mathcal{C}}(x_k, x_i; \epsilon, \alpha_0, d_0, k, l) = x_k x_i \int_0^{\alpha_0} \mathrm{d}\alpha \int_0^{\alpha_0} \mathrm{d}\beta\, \alpha^{-1-\epsilon}(1-\alpha)^{2d_0-1}$$
$$\times \beta^{-1-\epsilon}(1-\beta)^{2d_0-2+2\epsilon}[\alpha+(1-\alpha)(1-\beta)x_k]^{-1-\epsilon}[\beta+(1-\beta)x_i]^{-1-\epsilon}$$
$$\times \int_0^1 \mathrm{d}v \int_0^1 \mathrm{d}u\, v^{-\epsilon}(1-v)^{-\epsilon}u^{-\epsilon}(1-u)^{-\epsilon}$$
$$\times \left(\frac{\alpha+(1-\alpha)(1-\beta)x_k v}{2\alpha+(1-\alpha)(1-\beta)x_k}\right)^k \left(\frac{\beta+(1-\beta)x_i u}{2\beta+(1-\beta)x_i}\right)^l$$

where $k, l = -1, 0, 1, 2$. For this particular integral, we find

$$\mathcal{I}^{(4)}_{\mathcal{C}}(x_k, x_i; \epsilon, \alpha_0, d_0, k, l) = \frac{\delta_{k,-1}\delta_{l,-1}}{4\epsilon^4} - \left[\frac{\delta_{k,-1}\delta_{l,-1}}{2}\ln(x_k x_i)\right.$$
$$\left. + \frac{(1-\delta_{k,-1})\delta_{l,-1}}{2(1+k(1-\delta_{k,-1}))} + \frac{(1-\delta_{l,-1})\delta_{k,-1}}{2(1+l(1-\delta_{l,-1}))}\right]\frac{1}{\epsilon^3} + \mathrm{O}(\epsilon^{-2})\,.$$

The $\mathrm{O}(\epsilon^{-2})$ part is already rather cumbersome.

Phase space integrals - methods

Several different methods to compute the integrals have been explored

- use of IBPs to reduce to master integrals + solution of MIs by differential equations
- use of MB representations to extract pole structure + summation of nested series
- use of sector decomposition

Method of IBPs

1. Algebraic reduction of the integrand by means of partial fractioning

$$\frac{1}{x(1-x)(1-xyz)} = \frac{1}{x} + \frac{1}{1-yz}\frac{1}{1-x} + \frac{y^2z^2}{1-yz}\frac{1}{1-xyz}$$

   Note the appearance of a new denominator: $1-yz$. With increasing numbers of variables, the number of new denominators grows very rapidly.

2. Reduction to master integrals by means of IBP identities. We can use the standard Laporta algorithm to solve the IBP relations, but we find the occurrence of surface terms in the IBPs, consisting of integrals of lower dimensionality than the original ones.
3. Analytical evaluation of the master integrals. We obtain the ϵ expansion of the MIs by solving systems of differential equations, expanded in ϵ. The final results contain one- and two-dimensional harmonic polylogarithms. For some MIs, a nontrivial basis extension of 2dHPLs is necessary.

Method of MB representations

1. **Convert sums into products in the integrand**

$$\frac{1}{(a+b)^{\nu}} = \frac{1}{\Gamma(\nu)} \int_{q-\mathrm{i}\infty}^{q+\mathrm{i}\infty} \frac{\mathrm{d}z}{2\pi\mathrm{i}} a^{-\nu-z} b^{z} \Gamma(\nu+z)\Gamma(-\nu)$$

2. **Integrate over the real variables to obtain MB integrals**

$$(1-x)^{p} = \int_0^1 \mathrm{d}y\, y^{p} \delta(1-x-y)$$

$$\int_0^1 \mathrm{d}x\, \mathrm{d}y\, x^{p_1} y^{p_2} \delta(1-x-y) = \frac{\Gamma(p_1)\Gamma(p_2)}{\Gamma(p_1+p_2)}$$

3. **Resolve the pole structure by shifting integration contours.**
4. **Compute the MB integrals, converting them into sums over residua.**
5. **Perform the sums.**

Method of sector decomposition

Example

1. Transform the integral so that the range of integration is the unit hypercube, and all singularities are at the borders.

$$I = \int_0^1 \mathrm{d}x\,\mathrm{d}y\, x^{-1-\epsilon} y^{-\epsilon} [x + (1-x)y]^{-1}$$

2. Decompose into "sectors" using $1 = [\Theta(x-y) + \Theta(y-x)]$
3. Remap each integration region to the unit hypercube: for $x \geq y$ set $y \to xt$, for $y \geq x$ set $x \to yt$.

$$I = \int_0^1 \mathrm{d}x\,\mathrm{d}t\, x^{-1-2\epsilon} t^{-\epsilon} [1 + (1-x)t]^{-1}$$
$$+ \int_0^1 \mathrm{d}t\,\mathrm{d}y\, t^{-1-\epsilon} y^{-1-2\epsilon} [1 + (1-y)t]^{-1}$$

4. Resolve the pole structure using simple residuum subtraction. This gives a finite integral representation for the expansion coefficients.
5. Integrate these representations.

Analytical and numerical evaluation of the integrated counterterms

- To prove the cancellation of all IR poles requires, as a matter of principle, that all integrated counterterms are computed analytically.
- Analytical results are very fast and accurate compared to numerical ones.

HOWEVER

- They also show (in all cases where they are available) that the integrated counterterms are very smooth functions of the kinematic variables.

HENCE

- The final results for the integrated counterterms can be conveniently given by interpolating tables computed once and for all. Thus, for practical purposes, an efficient implementation is possible even in cases where the full analytical calculation is not feasible or practical (e.g. finite parts of integrated counterterms).

Overview of methods

Method	Analytical	Numerical
IBP	✔ Singly-unresolved integrals ✘ Bottleneck is the proliferation of denominators	✔ By evaluating full analytical results ✘ No numbers without full analytical results
MB	✔ Iterated singly-unresolved integrals ✘ Bottleneck is the evaluation of sums	✔ Direct numerical evaluation of MB integrals possible ✔ Fast and accurate
SD	✔ Easy to automatize ✘ Except for lowest order poles, possible only in principle	✘ Numerical behaviour is generally worse than MB method (speed, accuracy)

Results

Structure of the integrated counterterm

After summing over unresolved flavours ("counting of symmetry factors"), the integrated iterated singly-unresolved counterterm is a product of an insertion operator times the Born cross section

$$\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}} = \mathrm{d}\sigma_m^{\mathrm{B}} \otimes \mathbf{I}_{12}^{(0)}(\{p\}_m;\epsilon)$$

The insertion operator has the following structure in colour $\otimes$ flavour space

$$\begin{aligned}\mathbf{I}_{12}^{(0)}(\{p\}_m;\epsilon) = \left[\frac{\alpha_{\mathrm{s}}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 \Bigg\{ &\sum_i \left[\mathrm{C}_{12,f_i}^{(0)} \mathbf{T}_i^2 + \sum_k \mathrm{C}_{12,f_i f_k}^{(0)} \mathbf{T}_k^2\right] \mathbf{T}_i^2 \\ &+ \sum_{j,l} \left[\mathrm{S}_{12}^{(0),(j,l)} C_{\mathrm{A}} + \sum_i \mathrm{CS}_{12,f_i}^{(0),(j,l)} \mathbf{T}_i^2\right] \mathbf{T}_j \mathbf{T}_l \\ &+ \sum_{i,k,j,l} \mathrm{S}_{12}^{(0),(i,k)(j,l)} \{\mathbf{T}_i \mathbf{T}_k, \mathbf{T}_j \mathbf{T}_l\} \Bigg\}\end{aligned}$$

Here the $\mathrm{C}_{12,f_i}^{(0)}$, $\mathrm{C}_{12,f_i f_k}^{(0)}$ etc. functions depend on ϵ (having poles up to $\mathrm{O}(\epsilon^{-4})$) and kinematics (also on the factorized PS cut parameters).

The insertion operator

Example ($e^+e^- \to 2j$)

The Born matrix element is $|\mathcal{M}_2^{(0)}(1_q, 2_{\bar{q}})|^2$. Colour and kinematics is trivial

$$\mathbf{T}_1^2 = \mathbf{T}_2^2 = -\mathbf{T}_1\mathbf{T}_2 = C_\mathrm{F}\,, \qquad y_{12} = \frac{2p_1\cdot p_2}{Q^2} = 1$$

We find the insertion operator

$$\mathbf{I}_{12}^{(0)}(p_1, p_2; \epsilon) =$$

$$= \left[\frac{\alpha_\mathrm{s}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 \Bigg\{ \frac{2C_\mathrm{F}(3C_\mathrm{F} - C_\mathrm{A})}{\epsilon^4} + \frac{C_\mathrm{F}}{6}\Big[20C_\mathrm{A} + 81C_\mathrm{F} - 4T_\mathrm{R}n_\mathrm{f}$$

$$+ 12(3C_\mathrm{A} - 2C_\mathrm{F})\Sigma(y_0, D_0') + 12(2C_\mathrm{A} - C_\mathrm{F})\Sigma(y_0, D_0' - 1)\Big]\frac{1}{\epsilon^3} + \mathrm{O}(\epsilon^{-2})\Bigg\}$$

Notice the dependence on the factorized PS cut parameters y_0 and D_0',

$$\Sigma(z, N) = \ln z - \sum_{k=1}^{N} \frac{1-(1-z)^k}{k}$$

which should cancel between the various integrated counterterms in the full doubly-virtual contribution.

The insertion operator

Example ($e^+e^- \to 3j$)

The Born matrix element is $|\mathcal{M}_3^{(0)}(1_q, 2_{\bar{q}}, 3_g)|^2$. Colour is still trivial

$$\mathbf{T}_1^2 = \mathbf{T}_2^2 = C_\mathrm{F}\,, \quad \mathbf{T}_3^2 = C_\mathrm{A}\,, \quad \mathbf{T}_1\mathbf{T}_2 = \frac{C_\mathrm{A} - 2C_\mathrm{F}}{2}\,, \quad \mathbf{T}_1\mathbf{T}_3 = \mathbf{T}_2\mathbf{T}_3 = -\frac{C_\mathrm{A}}{2}$$

We find the insertion operator

$$\begin{aligned}
&\mathbf{I}_{12}^{(0)}(p_1, p_2, p_3; \epsilon) = \\
&\quad = \left[\frac{\alpha_\mathrm{s}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 \left\{ \frac{C_\mathrm{A}^2 + 2C_\mathrm{A}C_\mathrm{F} + 6C_\mathrm{F}^2}{\epsilon^4} + \left[\frac{11C_\mathrm{A}^2}{2} + \frac{50C_\mathrm{A}C_\mathrm{F}}{3} + 12C_\mathrm{F}^2\right.\right. \\
&\quad - \frac{C_\mathrm{A}T_\mathrm{R}n_\mathrm{f}}{3} - \frac{C_\mathrm{A}^2 T_\mathrm{R}n_\mathrm{f}}{C_\mathrm{F}} - 4C_\mathrm{F}T_\mathrm{R}n_\mathrm{f} + \left(\frac{5C_\mathrm{A}^2}{2} - C_\mathrm{A}C_\mathrm{F} - 8C_\mathrm{F}^2\right) \ln y_{12} \\
&\quad - \frac{C_\mathrm{A}(5C_\mathrm{A} + 8C_\mathrm{F})}{2}(\ln y_{13} + \ln y_{23}) + (C_\mathrm{A}^2 + 6C_\mathrm{A}2C_\mathrm{F} - 4C_\mathrm{F}^2)\Sigma(y_0, D_0') \\
&\quad \left.\left. + 4C_\mathrm{F}(C_\mathrm{A} - C_\mathrm{F})\Sigma(y_0, D_0' - 1)\right]\frac{1}{\epsilon^3} + \mathrm{O}(\epsilon^{-2})\right\}
\end{aligned}$$

Higher order expansion coefficients (in ϵ) are computed numerically.

Conclusions

Conclusions

- ✔ We have set up a general subtraction scheme for computing NNLO jet cross sections, for processes with no coloured particles in the initial state.
- ✔ We have investigated various methods to compute the integrated counterterms.
- ✔ We used the MB method to perform the integration of the iterated singly-unresolved counterterm, discussed in this talk. The SD method was used to provide independent checks.
- ✔ The integration of all singly-unresolved counterterms is finished. The iterated singly-unresolved counterterm is essentially finished.
- ✘ The integration of the doubly-unresolved counterterm is feasible with our methods, and is work in progress.